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§12.6 Quadratic Surfaces

Ex: Understand the surface w/ equation $x^2 + y^2 - 2x - 6y - z + 10 = 0$

Solution: First we will rewrite the equation (via completing the square)

$$x^2 + y^2 - 2x - 6y - z + 10 = 0$$

$$(x^2 - 2x) + (y^2 - 6y) - z + 10 = 0$$

$$(x^2 + 2(-1)x + (-1)^2 \cdot (-1)) + (y^2 + 2(-3)y + (-3)^2 - (-3)^2) - z + 10 = 0$$

$$(x-1)^2 - (-1)^2 + (y-3)^2 - (-3)^2 - z + 10 = 0$$

$$(x-1)^2 + (y-3)^2 - z = 0$$

Now we analyze the equation via cross-section
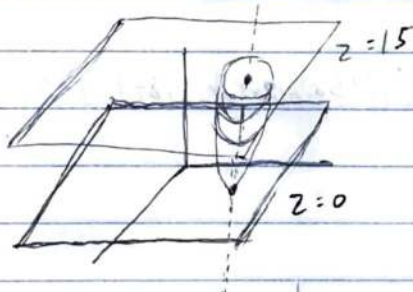when $z = k$ ← (constant)

$$(x-1)^2 + (y-3)^2 - k = 0$$

i.e. $(x-1)^2 + (y-3)^2 = k$ ← (elipse, or point, or empty if k is negative)

when $y = k$: $(x-1)^2 + (k-3)^2 - z = 0$

i.e. $z = (x-1)^2 + (k-3)^2$ ← parabola (upward facing)

when $x = k$: $(k-1)^2 + (y-3)^2 - z = 0$

i.e. $z = (k-1)^2 + (y-3)^2$ ← parabola (upward facing)

$z = 15$

$z = 0$

*Conic Sections

Ellipses $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Parabolas $\dfrac{x^2}{a^2} + \dfrac{y}{c} = 0$

Hyperbolas $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$

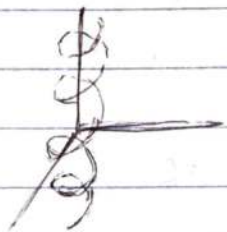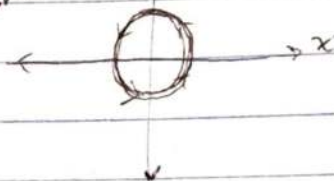| Equations	Name:
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z}{c} = 0$	Elliptic Paraboloid
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$	Ellipsoid
$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} - \dfrac{z}{c} = 0$	Hypebolic Paraboloid
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$	One-sheet Hyperboloid
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 0$	Cone
$-\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$	Two-sheet Hyperbola

§13.1: Space Curves

A __space curve__ is a function $\vec{r}: I \to \mathbb{R}^n$

Ex: The helix is the curve
$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

shadow:

Defn: The limit of space curve $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ at time $t = a$ is the component limit provided each component limits as $t \to a$

i.e. $\displaystyle\lim_{t \to a} \vec{r}(t) = \lim_{t \to a} \langle x(t), y(t), z(t) \rangle$

$$= \langle \lim_{t \to a} x(t), \lim_{t \to a} y(t), \lim_{t \to a} z(t) \rangle$$

~~illegible scribbled out text~~

__Exercise:__ Compute $\displaystyle\lim_{t \to \frac{7\pi}{16}} \vec{r}(t) = \langle (1 + 5\sin(20t))\cos(8t), (1 + 5\sin(20t))\sin(8t), \cos(20t) \rangle$